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Radiative leptonic decays of heavy mesons in heavy quark limit

C.-W. Hwang[1,2,a]

[1] Department of Physics, National Kaohsiung Normal University, Kaohsiung 824, Taiwan
[2] Physics Division, National Center for Theoretical Sciences (South), Tainan 701, Taiwan

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Abstract. We study the radiative leptonic decays of heavy mesons within the covariant light-front model. Using this model, both the form factors F_{V} and F_{A} have the same form when the heavy quark limit is taken. In addition, the relation between the form factor F_{V} and the decay constant of a heavy meson F_{H} is obtained. The hadronic parameter β can be determined by the parameters appearing in the wave function of the heavy meson. We find that the value of β is not only quite smaller than the one in the non-relativistic case, but also insensitive to the value of the light quark mass m_{q}. These results mean that the relativistic effects are very important in this work. We also obtain that the branching ratio of $B \to l\nu_l\gamma$ is about $(1.40–1.67)\times 10^{-6}$, in agreement with the general estimates in the literature.

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1 Introduction

The understanding of the strong and weak interactions of a heavy quark system is an important topic, and the purely leptonic decays $B \to l\nu_l$ seem to be the useful tools for this purpose. In particular, these processes are very simple in that no hadrons and photons appear in the final states. However, the rates of these purely leptonic decays are helicity suppressed with the factor of M_l^2/M_B^2 for $l = e$ and μ (in the τ channel, in spite of no suppression, it is hard to observe the decay because of the low efficiency). Therefore, it is natural to extend the purely leptonic B decay searches to the corresponding radiative modes $B \to l\nu_l\gamma$. These radiative leptonic decays receive two types of contributions: inner bremsstrahlung (IB) and structure dependent (SD) [1, 2]. As is known, the IB contributions are still helicity suppressed, while the SD ones are reduced by the fine structure constant α but they are not suppressed by the lepton mass. Accordingly, the radiative leptonic B decay rates could have an enhancement with respect to the purely leptonic ones, and would offer useful information about the Cabibbo–Kobayashi–Maskawa matrix element $|V_{\mathrm{ub}}|$ and the decay constant f_B [3]. Recently, there has been a great deal of theoretical attention [2–9] to these radiative leptonic B decays. Experimentally, the current upper limits for these modes are $Br(B^+ \to \mathrm{e}^+\nu_{\mathrm{e}}\gamma) < 2.2\times 10^{-5}$ and $Br(B^+ \to \mu^+\nu_\mu\gamma) < 2.3\times 10^{-5}$ at the 90% confidence level [10]. With the better statistics expected from the B factories, the observations of these decays could soon become feasible.

[a] e-mail: t2732@nknucc.nknu.edu.tw

The hadronic matrix elements responsible for the above decays can be calculated in various quark models. However, the relativistic effects must be considered seriously in the calculations, as the recoil momentum is large. This problem is taken into account by the light-front quark model (LFQM) [11], which has been considered as one of the best effective relativistic quark models in the description of the exclusive heavy hadron decays [12]. Its simple expression, relativistic structure, and predictive power have made possible wide applications of the LFQM in exploring and predicting the intrinsic heavy hadron dynamics. However, almost all the previous investigations have not covariantly extracted the form factors from the relevant matrix elements and paid enough attention to the consistency with heavy quark symmetry (HQS) and heavy quark effective theory (HQET). The covariant light-front model [13] has resolved the above-mentioned shortcomings in the LFQM and has improved the current understanding of the QCD analysis of heavy hadrons. This model consists of a heavy meson bound state in the heavy quark limit (namely $m_{\mathrm{Q}} \to \infty$), which is fully consistent with HQS, plus a reliable approach from this bound state to systematically calculate the $1/m_{\mathrm{Q}}$ corrections within HQET in terms of the $1/m_{\mathrm{Q}}$ expansion of the fundamental QCD theory. In this paper, we will use the covariant light-front model to investigate the radiative leptonic B decays in the heavy quark limit.

The paper is organized as follows. In Sect. 2 a general construction of covariant light-front bound states is provided; the diagrammatic rules within this model are also listed. In Sect. 3 we evaluate the decay constant of a heavy meson F_{H} and the form factors of the radiative leptonic heavy meson decay $F_{\mathrm{V,A}}$ in a completely covariant way. In Sect. 4 the relation between F_{H} and F_{V} is obtained. A few numerical calculations made with the help of the Gaussian-type wave function are presented. Finally, a summary is given in Sect. 5.

2 Covariant light-front model

The light-front bound states of heavy mesons that are written in a form exhibiting explicitly the boost covariance have been shown in the literature [12]. In this paper, we focus on the bound states of heavy mesons in the heavy quark limit:

$$\begin{aligned}|H(v;S,S_{\mathrm{z}})\rangle = \int &[\mathrm{d}^3k][\mathrm{d}^3p_{\mathrm{q}}]2(2\pi)^3v^+\delta^3(\overline{\Lambda}v-k-p_{\mathrm{q}})\\ &\times\sum_{\lambda_{\mathrm{Q}},\lambda_{\mathrm{q}}}R^{SS_{\mathrm{z}}}(X,\kappa_\perp,\lambda_{\mathrm{Q}},\lambda_{\mathrm{q}})\Phi^{SS_{\mathrm{z}}}(X,\kappa_\perp^2)\\ &\times b_v^\dagger(k,\lambda_{\mathrm{Q}})\mathrm{d}_{\mathrm{q}}^\dagger(p_{\mathrm{q}},\lambda_{\mathrm{q}})|0\rangle\,, \end{aligned}\tag{1}$$

where v^μ ($v^2=1$) is the velocity of the heavy meson, $k=p_{\mathrm{Q}}-m_{\mathrm{Q}}v$ is the residual momentum of a heavy quark, p_{q} is the momentum of a light antiquark,

$$[\mathrm{d}^3k]=\frac{\mathrm{d}k^+d^2k_\perp}{2(2\pi)^3v^+},\quad [\mathrm{d}^3p_{\mathrm{q}}]=\frac{\mathrm{d}p_{\mathrm{q}}^+\mathrm{d}^2p_{\mathrm{q}\perp}}{2(2\pi)^3p_{\mathrm{q}}^+}\,,\tag{2}$$

and $\overline{\Lambda}=M_{\mathrm{H}}-m_{\mathrm{Q}}$ is the residual center mass of a heavy meson. The relative momentum X was first introduced in [14] as the product of the longitudinal momentum fraction x of the valence antiquark and the mass of a heavy meson M_{H}, namely $X=xM_{\mathrm{H}}$. The relative transverse and longitudinal momenta, $\kappa_\perp$ and κ_{z}, are obtained by

$$\kappa_\perp=p_{\mathrm{q}\perp}-Xv_\perp,\quad \kappa_{\mathrm{z}}=\frac{X}{2}-\frac{m_{\mathrm{q}}^2+p_{\mathrm{q}\perp}^2}{2X}\,.\tag{3}$$

In (1), λ_{Q} and λ_{q} are helicities of a heavy quark and a light antiquark, respectively. In phenomenological calculations, one usually ignores the dynamical dependence of the light-front spin so that the function $R^{S,S_{\mathrm{z}}}$ can be approximately expressed by taking the covariant form for the so-called Melosh matrix [15] in the heavy quark limit,

$$R^{SS_{\mathrm{z}}}(X,\kappa_\perp,\lambda_{\mathrm{Q}},\lambda_{\mathrm{q}})=\begin{cases}\frac{1}{2}\sqrt{\frac{1}{v\cdot p_{\mathrm{q}}+m_{\mathrm{q}}}}\,\overline{u}_{\lambda_{\mathrm{Q}}}(v)(\mathrm{i}\gamma^5)v_{\lambda_{\mathrm{q}}}(p_{\mathrm{q}})\\ \text{for}\quad S=0,\\ \frac{1}{2}\sqrt{\frac{1}{v\cdot p_{\mathrm{q}}+m_{\mathrm{q}}}}\,\overline{u}_{\lambda_{\mathrm{Q}}}(v)(-\not{\epsilon})v_{\lambda_{\mathrm{q}}}(p_{\mathrm{q}})\\ \text{for}\ \ S=1\,,\end{cases}\tag{4}$$

where $u_{\lambda_{\mathrm{Q}}}(v)$ and $v_{\lambda_{\mathrm{q}}}(p_{\mathrm{q}})$ are spinors for the heavy quark and light antiquark,

$$\sum_\lambda u_\lambda(v)\overline{u}_\lambda(v)=\not{v}+1,\quad \sum_\lambda v_\lambda(p_{\mathrm{q}})\overline{v}_\lambda(p_{\mathrm{q}})=\not{p}_{\mathrm{q}}-m_{\mathrm{q}}\,.\tag{5}$$

The operators $b_{\mathrm{v}}^\dagger(k,\lambda_{\mathrm{Q}})$ and $d_{\mathrm{q}}^\dagger(p_{\mathrm{q}},\lambda_{\mathrm{q}})$ create a heavy quark and a light antiquark with

$$\begin{aligned}&\{b_{\mathrm{v}}(k,\lambda_{\mathrm{Q}}),\ b_{v'}^\dagger(k',\lambda'_{\mathrm{Q}})\}=2(2\pi)^3v^+\delta_{vv'}\delta^3(k-k')\delta_{\lambda_{\mathrm{Q}}\lambda'_{\mathrm{Q}}}\,,\\ &\{d_{\mathrm{q}}(p_{\mathrm{q}},\lambda_{\mathrm{q}}),\ d_{\mathrm{q}}^\dagger(p'_{\mathrm{q}},\lambda'_{\mathrm{q}})\}=2(2\pi)^3p_{\mathrm{q}}^+\delta^3(p_{\mathrm{q}}-p'_{\mathrm{q}})\delta_{\lambda_{\mathrm{q}}\lambda'_{\mathrm{q}}}\,.\end{aligned}\tag{6}$$

The normalization of the heavy meson bound states in the heavy quark limit is then given by

$$\begin{aligned}&\langle H(v',S',S'_z)|H(v,S,S_{\mathrm{z}})\rangle\\ &\quad=2(2\pi)^3v^+\delta^3(\overline{\Lambda}v'-\overline{\Lambda}v)\delta_{SS'}\delta_{S_zS'_z}\,,\end{aligned}\tag{7}$$

which leads to two things: first, the heavy meson bound state $|H(v;S,S_{\mathrm{z}})\rangle$ in this model rescales the one $|\tilde{H}(P;S,S_{\mathrm{z}})\rangle$ in the LFQM by $|\tilde{H}\rangle=\sqrt{M_{\mathrm{H}}}|H\rangle$ and, second, the space part $\Phi^{SS_Z}(X,\kappa_\perp^2)$ (called the light-front wave function) in (1) has the following wave function normalization condition:

$$\int\frac{\mathrm{d}X\mathrm{d}^2\kappa_\perp}{2(2\pi)^3X}|\Phi^{SS_{\mathrm{z}}}(X,\kappa_\perp^2)|^2=1\,.\tag{8}$$

In principle, the heavy quark dynamics is completely described by HQET, which is given by the $1/m_{\mathrm{Q}}$ expansion of the heavy quark QCD Lagrangian

$$\mathcal{L}=\overline{Q}(\mathrm{i}\not{D}-m_{\mathrm{Q}})Q=\sum_{n=0}^{\infty}\left(\frac{1}{2m_{\mathrm{Q}}}\right)^n\mathcal{L}_n\,.\tag{9}$$

Therefore, $|H(v;S,S_{\mathrm{z}})\rangle$ and $\Phi^{SS_{\mathrm{z}}}(X,\kappa_\perp^2)$ are then determined by the leading Lagrangian $\mathcal{L}_0$. Cheng et al. [13] have shown from the light-front bound state equation that $\Phi^{SS_{\mathrm{z}}}(X,\kappa_\perp^2)$ must be degenerate for $S=0$ and $S=1$. As a result, we can simply write

$$\Phi^{SS_{\mathrm{z}}}(X,\kappa_\perp^2)=\Phi(X,\kappa_\perp^2)\tag{10}$$

in the heavy quark limit. Equation (1) together with (4) and (10) is then the heavy meson light-front bound states in the heavy quark limit that obey HQS. Furthermore, (1) can be rewritten in a fully covariant form if $\Phi(X,\kappa_\perp^2)$ is a function of vp_{q}:

$$\Phi(X,\kappa_\perp^2)\longrightarrow\Phi(v\cdot p_{\mathrm{q}})\,,\tag{11}$$

where the antiquark q in bound states is on-mass-shell, $p_{\mathrm{q}}^-=(p_{\mathrm{q}\perp}^2+m_{\mathrm{q}}^2)/p_{\mathrm{q}}^+$. Hence,

$$v\cdot p_{\mathrm{q}}=\frac{1}{2X}\left(\kappa_\perp^2+m_{\mathrm{q}}^2+X^2\right).\tag{12}$$

As to the normalization condition of $\Phi(v\cdot p_{\mathrm{q}})$, (8) can also be rewritten in a covariant form:

$$\int\frac{\mathrm{d}^4p_{\mathrm{q}}}{(2\pi)^4}(2\pi)\delta(p_{\mathrm{q}}^2-m_{\mathrm{q}}^2)|\Phi(v\cdot p_{\mathrm{q}})|^2=1\,.\tag{13}$$

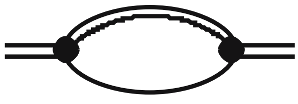


Fig. 1. A diagrammatic form for the heavy meson state normalization

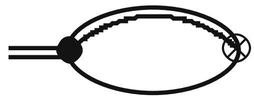


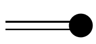
Fig. 2. The diagram for heavy meson decays

The left-hand side of (13) can be easily obtained in a diagrammatic way as shown in Fig. 1:

$$\begin{aligned}\text{Fig. } 1 &= \int \frac{\mathrm{d}^4 p_\mathrm{q}}{(2\pi)^4}(2\pi)\delta(p_\mathrm{q}^2-m_\mathrm{q}^2)|\Phi(v\cdot p_\mathrm{q})|^2\\ &\quad\times\frac{\mathrm{Tr}\Big[\Gamma_\mathrm{H}(1+\not v)\Gamma_\mathrm{H}(\not p_\mathrm{q}-m_\mathrm{q})\Big]}{4(v\cdot p_\mathrm{q}+m_\mathrm{q})}\\ &=(13)\,. \end{aligned}\tag{14}$$

In general, the on-shell Feynman rules within this model are given as follows [13]:

(i) The heavy meson bound state in the heavy quark limit gives a vertex as follows:

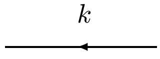


$$: \ \frac{1}{2}\sqrt{\frac{1}{v\cdot p_\mathrm{q}+m_\mathrm{q}}}\,\Phi(v\cdot p_\mathrm{q})\Gamma_\mathrm{H}\,. \tag{15}$$

(ii) The internal line attached to the bound state gives an on-shell propagator:

k

$$: \ 1+\not v \ \text{(for heavy quarks)}, \tag{16}$$



p_q

$$: \ \not p_\mathrm{q}-m_\mathrm{q} \ \text{(for light antiquarks)}\,. \tag{17}$$

(iii) For the internal antiquark line attached to the bound state, we sum over helicity and integrate the internal momentum using

$$\int \frac{\mathrm{d}^4 p_\mathrm{q}}{(2\pi)^4}(2\pi)\delta(p_\mathrm{q}^2-m_\mathrm{q}^2)\,. \tag{18}$$

(iv) For all other lines and vertices that do not attach to the bound states, the diagrammatic rules are the same as the Feynman rules in the conventional field theory.

3 Decay constants and form factors

Now, we shall present the evaluations of the decay constants and the form factors for heavy mesons within the covariant light-front model. First, the decay constants of pseudoscalar and vector mesons are defined by $\langle 0|\overline{q}\gamma^\mu\gamma_5 Q|\tilde{H}\rangle = \mathrm{i}f_\mathrm{H}p^\mu$ and $\langle 0|\overline{q}\gamma^\mu Q|\tilde{H}^*\rangle = f_{\mathrm{H}^*}M_{\mathrm{H}^*}\epsilon^\mu$, where q and Q are the light and heavy quark field operators, respectively. In the heavy quark limit, the decay constants have the expressions

$$\begin{aligned}\langle 0|\overline{q}\gamma^\mu\gamma_5 h_v|H(v)\rangle &= \mathrm{i}F_\mathrm{H}v^\mu\,,\\ \langle 0|\overline{q}\gamma^\mu h_v|H^*(v,\epsilon)\rangle &= F_{\mathrm{H}^*}\epsilon^\mu\,,\end{aligned}\tag{19}$$

so that

$$F_\mathrm{H} = f_\mathrm{H}\sqrt{M_\mathrm{H}}\,,\quad F_{\mathrm{H}^*} = f_{\mathrm{H}^*}\sqrt{M_{\mathrm{H}^*}}\,. \tag{20}$$

Using the above bound states and Feynman rules, it is very simple to evaluate the relevant matrix elements (diagrammatically shown in Fig. 2):

$$\langle 0|\overline{q}\gamma^\mu\gamma_5 h_v|H(v)\rangle = -\mathrm{i}\mathrm{Tr}\Big\{\gamma^\mu\gamma_5\frac{\not v+1}{2}\gamma_5\mathcal{M}_1\Big\}\,, \tag{21}$$

$$\langle 0|\overline{q}\gamma^\mu h_v|H^*(v,\epsilon)\rangle = \mathrm{Tr}\Big\{\gamma^\mu\frac{\not v+1}{2}\,\not\epsilon\mathcal{M}_1\Big\}\,, \tag{22}$$

where

$$\begin{aligned}\mathcal{M}_1 &= \sqrt{N_\mathrm{c}}\int\frac{\mathrm{d}^4 p_\mathrm{q}}{(2\pi)^4}(2\pi)\delta(p_\mathrm{q}^2-m_\mathrm{q}^2)\frac{\Phi(v\cdot p_\mathrm{q})}{\sqrt{v\cdot p_\mathrm{q}+m_\mathrm{q}}}(m_\mathrm{q}-\not p_\mathrm{q})\\ &= A_1+B_1\not v\,,\end{aligned}\tag{23}$$

and

$$A_1 = \sqrt{N_\mathrm{c}}\int\frac{\mathrm{d}^4 p_\mathrm{q}}{(2\pi)^4}(2\pi)\delta(p_\mathrm{q}^2-m_\mathrm{q}^2)\frac{\Phi(v\cdot p_\mathrm{q})}{\sqrt{v\cdot p_\mathrm{q}+m_\mathrm{q}}}\,m_\mathrm{q}\,, \tag{24}$$

$$B_1 = -\sqrt{N_\mathrm{c}}\int\frac{\mathrm{d}^4 p_\mathrm{q}}{(2\pi)^4}(2\pi)\delta(p_\mathrm{q}^2-m_\mathrm{q}^2)\frac{\Phi(v\cdot p_\mathrm{q})}{\sqrt{v\cdot p_\mathrm{q}+m_\mathrm{q}}}\,v\cdot p_\mathrm{q}\,. \tag{25}$$

Here $N_\mathrm{c}=3$ is the number of colors. Thus, it is easily found that

$$\begin{aligned}F_\mathrm{H} &= 2(A_1-B_1)\\ &= 2\sqrt{N_\mathrm{c}}\int\frac{\mathrm{d}^4 p_\mathrm{q}}{(2\pi)^4}(2\pi)\delta(p_\mathrm{q}^2-m_\mathrm{q}^2)\\ &\quad\times\frac{\Phi(v\cdot p_\mathrm{q})}{\sqrt{v\cdot p_\mathrm{q}+m_\mathrm{q}}}(v\cdot p_\mathrm{q}+m_\mathrm{q}) = F_{\mathrm{H}^*}\,,\end{aligned}\tag{26}$$

as expected from HQS. Using (12), the integral in(26) gives

$$\begin{aligned}F_\mathrm{H} &= 2\sqrt{2N_\mathrm{c}}\int\frac{\mathrm{d}X\,\mathrm{d}^2\kappa_\perp}{2(2\pi)^3\sqrt{X}}\,\frac{\Phi(X,\kappa_\perp^2)}{\sqrt{\kappa_\perp^2+(m_\mathrm{q}+X)^2}}\\ &\quad\times\left(\frac{X}{2}+\frac{m_\mathrm{q}^2+\kappa_\perp^2}{2X}+m_\mathrm{q}\right).\end{aligned}\tag{27}$$

Next, the form factors for the radiative leptonic decays, which come from vector and axial vector currents, are defined by [5]

$$\langle\gamma(p_\gamma,\epsilon)|\bar q\gamma_\mu Q|\tilde{H}(P)\rangle = \mathrm{e}\frac{f_\mathrm{V}(q^2)}{M_\mathrm{H}}\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}P^\alpha p_\gamma^\beta\,, \tag{28}$$

$$\begin{aligned}\langle\gamma(p_\gamma,\epsilon)|\bar q\gamma_\mu\gamma_5 Q|\tilde{H}(P)\rangle &= \mathrm{ie}\frac{f_\mathrm{A}(q^2)}{M_\mathrm{H}}[\epsilon_\mu^*(P\cdot p_\gamma)\\ &\quad-(p_\gamma)_\mu(\epsilon^*\cdot P)]\,,\end{aligned}\tag{29}$$

respectively, where $q=P-p_\gamma$. If one ignores the lepton mass (namely ignores the IB contributions), the leading contributions to $f_{\rm V}(q^2)$ come from pole diagrams with one vector intermediate state ($J^P=1^-$), and those to $f_{\rm A}(q^2)$ from two axial vector states ($J^P=1^+$) [2]. In the non-relativistic quark model (NRQM), the dominant contribution comes from the former state,

$$f_V(q^2)=\frac{q_{\rm q}}{2(E_\gamma+\triangle)}\,\beta f_{H^*}M_{H^*}\,, \tag{30}$$

where $q_{\rm q}$ is the charge of a light quark in units of e, $\triangle=M_{H^*}-M_{\rm H}$, and the hadronic parameter $\beta\simeq 3\,{\rm GeV}^{-1}$ [16]. The contributions of the latter states are also proportional to $f^i_{\rm A}$, the axial vector meson decay constants. But these are zero because the wave function at the origin for an orbitally excited state vanishes.

In the heavy quark limit, the above form factors have the expressions

$$\langle\gamma(p_\gamma,\epsilon)|\bar q\gamma_\mu h_v|H(v)\rangle={\rm e}F_{\rm V}(E_\gamma)\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}v^\alpha p^\beta_\gamma, \tag{31}$$

$$\begin{aligned}\langle\gamma(p_\gamma,\epsilon)|\bar q\gamma_\mu\gamma_5 h_v|H(v)\rangle&={\rm ie}F_{\rm A}(E_\gamma)\\&\times[\epsilon^*_\mu(v\cdot p_\gamma)-(p_\gamma)_\mu(\epsilon^*\cdot v)]\,,\end{aligned} \tag{32}$$

where $E_\gamma=v\cdot p_\gamma$. So,

$$F_i=\frac{f_i}{\sqrt{M_{\rm H}}}\quad(\text{for}\quad i={\rm V, A}). \tag{33}$$

The contributions to these form factors coming from the coupling of the photon to the heavy and light quarks are diagrammatically shown in Fig. 3a and b, respectively. Using the above bound states and Feynman rules, the former contributions to the form factors, for example $F_{\rm V}$, are given by

$$\begin{aligned}\Gamma^{(Q)}_\mu&=q_{\rm Q}\frac{\rm ie}{2m_{\rm Q}v\cdot p_\gamma}{\rm Tr}\\&\times\left\{\gamma_\mu(m_{\rm Q}\not v-\not p_\gamma+m_{\rm Q})\not\epsilon\frac{\not v+1}{2}\gamma_5\mathcal{M}_1\right\},\end{aligned} \tag{34}$$

where $q_{\rm Q}$ is the charge of a heavy quark in units of e and $\mathcal{M}_1$ was obtained in (23). Performing the trace and comparing with (26), the heavy quark contribution to the form factor $F_{\rm V}$ is obtained by

$$F^{(Q)}_{\rm V}(E_\gamma)=\frac{q_{\rm Q}}{2E_\gamma}\frac{2(A_1-B_1)}{m_{\rm Q}}=\frac{q_{\rm Q}}{2E_\gamma}\frac{F_{\rm H}}{m_{\rm Q}}\,, \tag{35}$$

as obtained from perturbative QCD [8]. In the case of the light quark contribution, it can be written as

$$\Gamma^{(q)}_\mu={\rm i}q_{\rm q}{\rm Tr}\left\{\gamma_\mu\frac{\not v+1}{2}\gamma_5\mathcal{M}_2\not p_\gamma\not\epsilon\right\}, \tag{36}$$

where

$$\begin{aligned}\mathcal{M}_2&=\sqrt{N_{\rm c}}\int\frac{{\rm d}^4p_{\rm q}}{(2\pi)^4}(2\pi)\delta(p^2_{\rm q}-m^2_{\rm q})\frac{\Phi(v\cdot p_{\rm q})}{\sqrt{v\cdot p_{\rm q}+m_{\rm q}}}\\&\times\frac{m_{\rm q}-\not p_{\rm q}}{2p_\gamma\cdot p_{\rm q}}\\&=A_2+B_2\not v+C_2\not p_\gamma\,,\end{aligned} \tag{37}$$

and

$$A_2=\sqrt{N_{\rm c}}\int\frac{{\rm d}^4p_{\rm q}}{(2\pi)^4}(2\pi)\delta(p^2_{\rm q}-m^2_{\rm q})\frac{\Phi(v\cdot p_{\rm q})}{\sqrt{v\cdot p_{\rm q}+m_{\rm q}}}\frac{m_{\rm q}}{2p_\gamma\cdot p_{\rm q}}\,, \tag{38}$$

$$B_2=\sqrt{N_{\rm c}}\int\frac{{\rm d}^4p_{\rm q}}{(2\pi)^4}(2\pi)\delta(p^2_{\rm q}-m^2_{\rm q})\frac{\Phi(v\cdot p_{\rm q})}{\sqrt{v\cdot p_{\rm q}+m_{\rm q}}}\frac{1}{2v\cdot p_\gamma}\,, \tag{39}$$

$$\begin{aligned}C_2&=\sqrt{N_{\rm c}}\int\frac{{\rm d}^4p_{\rm q}}{(2\pi)^4}(2\pi)\delta(p^2_{\rm q}-m^2_{\rm q})\frac{\Phi(v\cdot p_{\rm q})}{\sqrt{v\cdot p_{\rm q}+m_{\rm q}}}\frac{1}{2v\cdot p_\gamma}\\&\times\frac{v\cdot p_{\rm q}v\cdot p_\gamma-p_{\rm q}\cdot p_\gamma}{v\cdot p_\gamma p_{\rm q}\cdot p_\gamma}\,.\end{aligned} \tag{40}$$

It is easily found that

$$\begin{aligned}F^{(q)}_{\rm V}&=2q_{\rm q}(A_2-B_2)\\&=\frac{q_{\rm q}}{2E_\gamma}\sqrt{N_{\rm c}}\int\frac{{\rm d}^4p_{\rm q}}{(2\pi)^4}(2\pi)\delta(p^2_{\rm q}-m^2_{\rm q})\frac{\Phi(v\cdot p_{\rm q})}{\sqrt{v\cdot p_{\rm q}+m_{\rm q}}}\\&\times 2\left(1+m_{\rm q}\frac{v\cdot p_\gamma}{p_{\rm q}\cdot p_\gamma}\right).\end{aligned} \tag{41}$$

Using the light-front relative momentum, the integral (41) gives

$$\begin{aligned}F^{(q)}_{\rm V}&=\frac{q_{\rm q}}{2E_\gamma}2\sqrt{2N_{\rm c}}\int\frac{{\rm d}X{\rm d}^2\kappa_\perp}{2(2\pi)^3\sqrt{X}}\frac{\Phi(X,\kappa^2_\perp)}{\sqrt{\kappa^2_\perp+(m_{\rm q}+X)^2}}\\&\times\left(1+\frac{m_{\rm q}}{m^2_{\rm q}+\kappa^2_\perp/X}\right).\end{aligned} \tag{42}$$

In this way, we also calculate the form factors $F^{(q,Q)}_{\rm A}$ which come from the coupling to the axial vector current and find that $F^{(q,Q)}_{\rm A}(E_\gamma)=F^{(q,Q)}_{\rm V}(E_\gamma)$. These results are consistent with those in [8], but contrary to those in NRQM [2]. In addition, it has been emphasized in the literature that various hadronic form factors calculated in the LFQM should

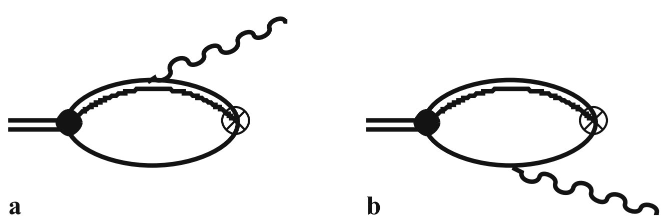


Fig. 3. The diagrams for the radiative leptonic decays of a heavy meson

be extracted only from the plus component of the corresponding currents. Also, the LFQM calculations for decay processes are restricted to a specific Lorentz frame (namely, zero momentum transfer). Now, we see that the covariant light-front model removes the above restrictions, straightforwardly extracts the hadronic form factors $F_{\mathrm{V,A}}$, and obtains the result $F_{\mathrm{H}} = F_{\mathrm{H}^*}$, which is consistent with HQS.

4 Numerical calculations and discussion

At first glance, the decay constant F_{H} in (27) does not seem to connect with $F_{\mathrm{V}}^{(q)}$ in (42). However, since the wave function $\Phi(X,\kappa_\perp^2)$ is a function of $v\cdot p_{\mathrm{q}}$ (see (11)) for a fully covariant bound state, and

$$v\cdot p_{\mathrm{q}} = E_{\mathrm{q}} = \sqrt{m_{\mathrm{q}}^2+\kappa_\perp^2+\kappa_{\mathrm{z}}^2}\,,$$

thus $\Phi(X,\kappa_\perp^2)$ is even in κ_{z}. It follows that

$$\int \frac{\mathrm{d}X\mathrm{d}^2\kappa_\perp}{2(2\pi)^3\sqrt{X}}\,\frac{\Phi(X,\kappa_\perp^2)}{\sqrt{\kappa_\perp^2+(m_{\mathrm{q}}+X)^2}}\,\kappa_{\mathrm{z}} = 0\,. \tag{43}$$

From (3) and (43), F_{H} and $F_{\mathrm{V}}^{(q)}$ are easily rewritten in the simpler forms

$$F_{\mathrm{H}} = 2\sqrt{2N_{\mathrm{c}}}\int \frac{\mathrm{d}X\mathrm{d}^2\kappa_\perp}{2(2\pi)^3\sqrt{X}}\,\frac{\Phi(X,\kappa_\perp^2)}{\sqrt{\kappa_\perp^2+(m_{\mathrm{q}}+X)^2}}(X+m_{\mathrm{q}})\,, \tag{44}$$

$$F_{\mathrm{V}}^{(q)} = \frac{q_{\mathrm{q}}}{2E_\gamma m_{\mathrm{q}}}2\sqrt{2N_{\mathrm{c}}}\int \frac{\mathrm{d}X\mathrm{d}^2\kappa_\perp}{2(2\pi)^3\sqrt{X}} \times \frac{\Phi(X,\kappa_\perp^2)}{\sqrt{\kappa_\perp^2+(m_{\mathrm{q}}+X)^2}}(X+m_{\mathrm{q}})\left(\frac{m_{\mathrm{q}}}{X}\right). \tag{45}$$

It is known that $Xv^+ = p_{\mathrm{q}}^+ = E_{\mathrm{q}}+\kappa_{\mathrm{z}}$, and therefore the ratio m_{q}/X in (45) will be equal to unity when the antiquark is at rest. Thus, in the non-relativistic case, (45) can be reduced to

$$F_{\mathrm{V}}^{(q)} = \frac{q_{\mathrm{q}}}{2E_\gamma}\frac{F_{\mathrm{H}}}{m_{\mathrm{q}}}\,. \tag{46}$$

From (46), one can extract the hadronic parameter $\beta_{\mathrm{NR}} = 1/m_{\mathrm{q}}$, which is consistent with the result of the NRQM [2, 16]. In the relativistic case, we can evaluate (45) directly to extract β_{R}. There are several popular phenomenological wave functions that have been employed to describe various hadronic structures in the literature. We choose the Gaussian-type wave function $\Phi_G(v\cdot p_{\mathrm{q}})$ [13], which has been widely used in the study of heavy mesons:

$$\Phi_G(v\cdot p_{\mathrm{q}}) = 4\left(\frac{\pi}{\omega^2}\right)^{3/4}\sqrt{v\cdot p_{\mathrm{q}}} \times\exp\left\{-\frac{1}{2\omega^2}\left[(v\cdot p_{\mathrm{q}})^2-m_{\mathrm{q}}^2\right]\right\}. \tag{47}$$

The parameters appearing in this wave function are m_{q} and ω. From the NRQM to the LFQM, the value of a light quark mass varies from 0.33 to 0.23 GeV. Thus, we take several different values of m_{q} and f_B to evaluate the parameter β_{R}. These results are listed in Table 1. We find that, in general, the values of β_{R} are not only quite smaller than the ones of β_{NR}, but also insensitive to the values of m_{q}. These results mean that the typical values of X in the integrations are much larger than m_{q}, that is to say, the relativistic effects are very important in these evaluations.

Table 1. Parameters m_{q} and ω fitted to the decay constant f_{B}. Also shown are the results for the hadronic parameters β_{R} and β_{NR}

f_{B} (GeV)		0.180			0.190	
m_{q} (GeV)	0.33	0.28	0.23	0.33	0.28	0.23
ω (GeV)	0.480	0.486	0.493	0.499	0.505	0.513
β_{R} (GeV^{-1})	1.58	1.66	1.73	1.55	1.61	1.67
β_{NR} (GeV^{-1})	3.03	3.57	4.35	3.03	3.57	4.35

This hadronic parameter β_{R} can be used to calculate the branching ratio of the B meson radiative decay. The decay rate for $B\to l\nu_l\gamma$ differential in the photon energy is given by

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma} = \frac{\alpha G_F^2|V_{\mathrm{ub}}|^2M_B^4}{48\pi^2}[f_V^2(E_\gamma)+f_A^2(E_\gamma)]y^3(1-y)\,, \tag{48}$$

where $y\equiv 2E_\gamma/M_B$. In the heavy quark limit, there is only one contribution $F_{\mathrm{V,A}}^{(q)}$ to $F_{\mathrm{V,A}}$; thus $f_{V,A}$ in (28) and (29) can be obtained by

$$f_{V,A} = \frac{q_{\mathrm{q}}}{2E_\gamma}\beta_{\mathrm{R}}f_BM_B\,. \tag{49}$$

Integrating the photon energy in (48), we can obtain the decay rate

$$\Gamma(B\to l\nu_l\gamma) = \frac{\alpha G_F^2|V_{\mathrm{ub}}|^2M_B^5}{288\pi^2}\,q_{\mathrm{q}}^2\beta_{\mathrm{R}}^2f_B^2\,. \tag{50}$$

Taking $f_B = 0.18$ GeV, $|V_{\mathrm{ub}}| = 3.33\times10^{-3}$ [17], and $\tau_{\mathrm{B}^+} = 1.67\times10^{-12}$ s [17], we obtain $Br(B\to l\nu_l\gamma) = (1.40–1.67)\times10^{-6}$. This result agrees well with those in [5] and [7], where the light cone QCD sum rules and the LFQM were used in their calculations, respectively. However, this result is about a factor of two smaller and larger than those in [6] and [3], respectively. The parameter β_{R} also relates to a ratio $Br(B\to l\nu_l\gamma)/Br(B\to\mu\nu_\mu)$. The pure leptonic decay rate is given by

$$\Gamma(B\to\mu\nu_\mu) = \frac{G_F^2|V_{\mathrm{ub}}|^2M_B^3}{8\pi^2}\,f_B^2\left(\frac{M_\mu^2}{M_B^2}\right)\left(1-\frac{M_\mu^2}{M_B^2}\right). \tag{51}$$

Taking the relevant values, we obtain the ratio

$$\frac{Br(B\to l\nu_l\gamma)}{Br(B\to\mu\nu_\mu)} \simeq 2.0\beta_{\mathrm{R}}^2 \simeq 5.0–6.0\,, \tag{52}$$

which is within the range of 1–30 as expected in [2].

5 Summary and perspective

In this work we have calculated the decay constants $F_{\mathrm{H,H}^*}$ for heavy mesons and the form factors $F_{\mathrm{V,A}}(E_\gamma)$ for the radiative leptonic decays $H \to l\nu\gamma$ within the covariant light-front approach. In accordance with HQS and the results of [8], $F_{\mathrm{H}} = F_{\mathrm{H}^*}$ and $F_{\mathrm{V}}(E_\gamma) = F_{\mathrm{A}}(E_\gamma)$ to the tree level, respectively. In addition, the form factor $F_{\mathrm{V}}(E_\gamma)$ can be related to the decay constant F_{H^*} by considering an 1^- intermediate state contribution. The relevant hadronic parameter β_{R}, in contrast to $\beta_{\mathrm{NR}} = 1/m_{\mathrm{q}}$ in the NRQM, has been determined by the parameters m_{q} and ω in this covariant model. The comparison between β_{R} and β_{NR} was listed quantitatively in Table 1. We conclude that the relativistic effects are quite obvious in these modes. We have also obtained $Br(B \to l\nu_l\gamma) = (1.40–1.67)\times 10^{-6}$ and $Br(B \to l\nu_l\gamma)/Br(B \to \mu\nu_\mu) = 5.0–6.0$, in agreement with the general estimates in the literature.

The covariant model removes some restrictions often occurring in the usual LFQM calculation and, as we have shown in this paper, allows one to perform some extremely simple evaluations of various heavy meson properties. Further applications to other properties of heavy mesons and physical processes will be presented in subsequent papers.

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